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CONTEXT OF THE INVENTORY MANAGEMENT EXPENSES IN THE CASE OF PLANNED SHORTAGES

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ABSTRACT

The main purpose of the paper is to present the relations between the different cost factors of the inventory management systems, and the context between the order quantities and the cost level. The theoretical approach of the model assumes a deterministic operational environment with planned shortages. We make the examination of the contexts by applying the *ceteris paribus* principle; we change only one cost factor from among the initial conditions at once and examine its effect on the cost level.

By using the economic order quantity with the planned shortage model, we can define the optimal order quantity, along which our stock management can be guaranteed by the most favourable cost level. The optimisation of the inventory level and the inventory management expenses together means an important factor in the competitiveness of the company. During the definition of the optimal inventory level of purchased parts, the purchasing and stock holding costs, and also the consequence of shortages play an important role. The presentation of the specific expense factors in each other's function, and the representation of the onetime order expenses show their proportion compared to each other and the effect of their change on the total cost, and define the opportunities of the optimisation. The significance of the model is that it represents the level line of costs, the movement of the different cost factors in relation to others and their operating mechanism. Thus, it facilitates the representation of costs and the definition of the direction of optimisation.

KEY WORDS

cost factor, optimum, order quantity, purchasing, shortage, stock holding

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INTRODUCTION

It often happens in practical logistics that the actual utilisation demand cannot be satisfied immediately. The continuity of service, in some cases, is broken by a disturbance in a stage of the supply chain, which causes a significant confusion for both the customer and the supplier. In other cases, the reason

is a planned stock management strategy that can be led back to a certain aspect of economic efficiency.

The classical stockpile management approaches the optimisation of the stock level from the side of expenses, meaning that the optimal stock level is represented by the stock derived from the lowest total costs. Among the costs of the stockpile management system, we differentiate three basic cost categories:

- cost elements related to the procurement activity,
- costs related to stock holding,
- costs related to stock shortage consequences (Krampe et al., 2012).

These three cost groups can be modified to the detriment of one another (Halászné, 1998; Kummer et al., 2009). The holding costs increase linearly with the increase in the lot size, while the costs related to procurement decrease with the increase in the order quantity (Tersine & Barman, 1991). Similarly, the holding costs are in a trade-off relation with the costs of stock shortage. The task is the definition of the optimum of the total costs function that fulfils the cost-minimising target, and the quantification of the derivable order quantity and of the order period (Koltai, 2009).

1. LITERATURE REVIEW

The first scientific model about an optimal lot size determination was published by Harris in 1913, in the article “How many parts to make at once” (Harris, 1913). This model defines the production quantity optimisation with not acceptable stock-out periods and assumes deterministic conditions. Several extensions of the basic Economic Order Quantity model are defined since that, describing the real operational processes more and more in details, and gives answers to the practical issues. In the case of deterministic inputs, the model is extended to the analysis of the deterioration of goods (Ghare & Scharder, 1963), the quantity discounts (Tersine & Barman, 1991), limited supplier capacity, the dynamic version of the economic lot size (Wagner & Whitin, 1958), etc., and some research has also focused on the direction of stochastic factors, such as the demand fluctuation, the lead time variation, the fraction of the defective items (Porteus, 1986) or shortages using a probability density function, etc. Chang and Dye applied the model for deteriorating items, where the time-varying demand is partial backlogged (Chang & Dye, 1999).

The initial assumption of Harris’s basic model that shortages are not allowed is too restrictive in real industrial working environment. Many researchers (Park, 1982; Hollier & Mak, 1983; Grubbström & Erdem, 1999) assume that during shortage periods all demand either backlogged or lost. Deb and Chaudhuri extended the economic order quantity model by including completely backordered shortages (Deb & Chaudhuri, 1987). They defined a replenishment policy where the inventory cycles were divided into

two periods, in the first one the demand is covered by inventory, while in the second part of the cycle it is followed by a period of shortages. In their model shortages were allowed in all cycles except the final one. Also Dave worked out a heuristic inventory-replenishment model with planned shortages and linearly increasing demand (Dave, 1989). Teng and Yang considered a partial backlogging rate during the stockout periods when demand and cost are fluctuating with time (Teng & Yang, 2004). Many researchers extended the planned shortage model by considering varying shortage cost values (Wee et al., 2007), and also assuming deteriorating items with imperfect quality (Salameh & Jaber, 2000; Wee et al., 2006; Eroglu & Ozdemir, 2007).

2. RESEARCH METHODS

By walking around the initial conditions of the economic order quantity model with planned shortages, we analyse the context of the inventory management expenses. We make the examination of the contexts by applying the *ceteris paribus* principle; we change only one cost factor from among the initial conditions at once. First we describe the context of the inventory cycle model with planned shortages, and define the calculations methods of the different cost factors. The main purpose of our analysis is to define the economic order quantity, the optimum backorder level and the optimum level of different cost factors, and also to present the relation between the cost factors. By introducing a multiplying factor derived from the relation between the costs of stock holding and stock shortage the optimal quantity analysis can be simplified to the basic model made up of one purchase and stock holding cost.

3. RESEARCH RESULTS

3.1. DETERMINATION OF THE ECONOMIC ORDER QUANTITY IN THE CASE OF A PLANNED SHORTAGE

As an initial condition, we define that the unsatisfied demand due to stock shortage can be rescheduled by a defined cost level, and it will be fully performed at a later date (Vörös, 1991). The main questions of stock management models are the optimum quantity that can be procured on one occasion

by most favourable total costs, and the optimal scheduling of procurement. The balance between the stock level and the costs can be defined using the economic order quantity with a planned shortage model, having the following initial conditions (Halász, 1998; Kummer et al., 2009; Krampe et al., 2012; Koltai, 2009; Vörös, 1991; Illés, 1998; Szegedi & Prezenszki, 2003):

- The supply rate can be considered being infinite; the stock replenishment is immediate, and so the replenishment time equals zero;
- The ordered quantity arrives as one item; frequency of supplies is scheduled for identical periods;
- The demand is known and pre-definable with absolute certainty;
- Both the customer and the supplier want to satisfy the demand. The demand is continuous, and the utilisation has a consistent intensity; thus, the demand rate is constant. Accordingly, within a supply period, the stock level shows a strictly monotonous descending linear function in relation to time;
- The stock shortage is accepted at a certain cost;
- The ordering costs are independent of the order quantity;
- The holding costs per unit are constant, and they change linearly with the stock quantity;

- By assuming an infinite time horizon, the costs are independent of the time factor;
- The purchase price per unit does not depend on quantity; thus, the purchase price does not influence the stock management policy to be chosen (Vörös, 2010; Chopra & Meindl, 2007).

In the case of a constant utilisation demand with continuous and uniform intensity (Fig. 1. a) and a procurement cycle with uniform period intervals, if the opening stock d of period t is smaller than the total utilisation demand q during the period, the stocks before the next period will decrease to zero at a certain t_0 point of time, followed by a stock shortage period with the t_2 length, at the end of which the stocks will be replenished. During the period t_2 , the continuous demand will lead to a backlog of the level s .

In one diagram, Figure 1. b) summarises the arrival of stocks and the development of demands in relation to time as a cumulated value. The difference between these two values shows the relation between the demands satisfied on time and the demands that are rescheduled.

The purchase costs incurring during the whole analysed period can be defined by multiplying the one-off purchase cost by the frequency of procurements (Kulcsár, 1998):

$$C_o = \frac{Q}{q} \cdot c_o \tag{1}$$

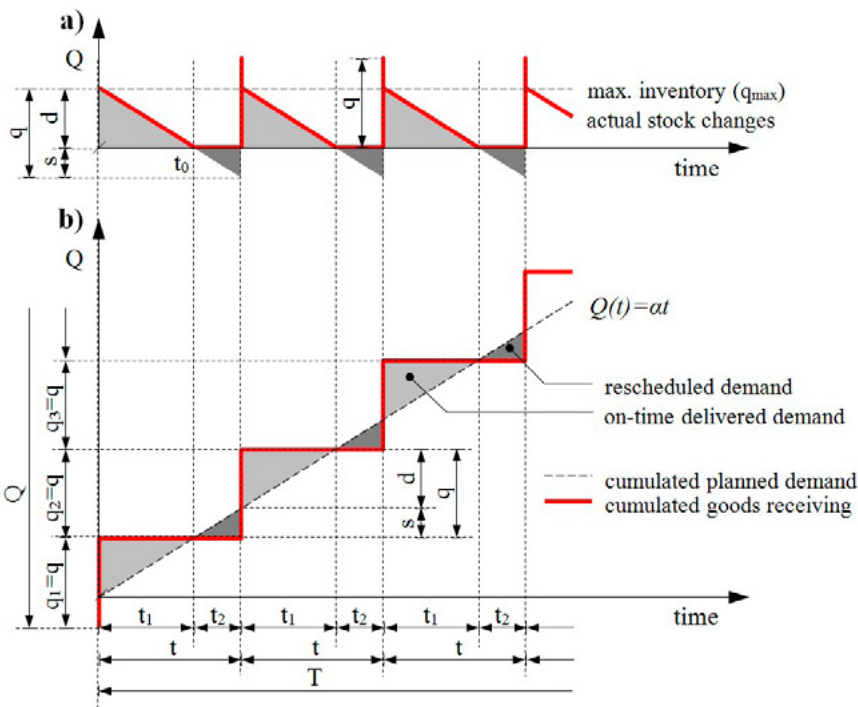


Fig. 1. Optimal stock level in the case of regular stock replenishment and demand with uniform intensity with periods of stock shortage

where:

- C_o – the total purchase cost for the examined period,
- Q – the total purchase demand for the examined period,
- q – the purchase demand for a single period, economic order quantity,
- c_o – the cost of a single purchase order.

The holding costs can be defined using the area of sections due to the t_1 period of the sawtooth diagram (Kulcsár, 1998):

$$C_h = \frac{1}{2} \cdot d \cdot t_1 \cdot \frac{Q}{q} \cdot c_h = \frac{1}{2} \cdot d^2 \cdot \frac{T}{q} \cdot c_h = \frac{d^2}{2 \cdot q} \cdot v \cdot r \quad (2)$$

where:

- C_h – the total holding cost for the examined period,
- d – the portion of the demand covered by stock within one single period,
- t_1 – the period, during the demand is performed without delay at the time of its occurrence,
- c_h – the holding cost per time unit,
- T – the length of the complete period,
- v – the purchasing price per unit,
- r – the annual holding cost rate.

During the quantification of the stock shortage costs, we must start from the relation that the continuous demand increases the level of the backlog, which can be expressed using the area of sections due to the period t_2 of the sawtooth diagram (Krampe et al., 2012; Kulcsár, 1998):

$$\begin{aligned} C_s &= \frac{1}{2} \cdot (q - d) \cdot t_2 \cdot \frac{Q}{q} \cdot c_s = \\ &= \frac{(q - d)^2}{2 \cdot q} \cdot T \cdot c_s = \frac{s^2}{2 \cdot q} \cdot T \cdot c_s \end{aligned} \quad (3)$$

where:

- C_s – the shortage cost during the whole analysed period,
- t_2 – the period, during which the demands due must be rescheduled for a later date,
- c_s – the shortage cost per time unit.

The basic model of the economic order quantity starts from the relation that the purchase cost, the holding cost changes, and the stock level change according to the order quantity. Accordingly, the more rarely orders are made, the more favourable the purchase costs are per unit, and at the same time, the holding costs are increasing linearly (Vijayan & Kumaran, 2009; Jaynes, 2003). The function of total

costs can be defined as the sum of these three costs and the value of the purchased parts. The objective function is defining the minimum of the function of total costs (Koltai, 2009; Kummer et al., 2009; Vörös, 2010; Krampe et al., 2012; Chopra & Meindl, 2007; Kulcsár, 1998):

$$\begin{aligned} C'(q; d) &= Q \cdot v + C_o + C_h + C_s = \\ &= \frac{Q}{q} \cdot c_o + \frac{d^2}{2 \cdot q} \cdot T \cdot c_h + \frac{(q - d)^2}{2 \cdot q} \cdot T \cdot c_s \rightarrow \min \end{aligned} \quad (4)$$

where:

- C' – the total cost of inventory management for the examined period, with the value of purchased items.

The optimal order quantity can be defined by solving the system of previous equations, where the form of partial derivatives according to q and d of the function of total costs is set equal to zero (Krampe et al., 2012; Vörös, 1991; Kulcsár, 1998; Huang & Wu, 2016; Paknejad et al., 2015; Cárdenas-Barrón, 2010):

$$q = d \cdot \frac{c_h + c_s}{c_s} = \sqrt{\frac{2 \cdot Q}{T} \cdot \frac{c_o}{c_h}} \cdot \sqrt{\frac{c_h + c_s}{c_s}} \quad (5)$$

The on-time delivered quantity can be calculated as followed:

$$d = \sqrt{\frac{2 \cdot Q}{T} \cdot \frac{c_o}{c_h}} \cdot \sqrt{\frac{c_s}{c_h + c_s}} \quad (6)$$

The optimal amount to be backordered (Vörös, 2010):

$$s = q - d = q \cdot \frac{c_h}{c_h + c_s} = \sqrt{\frac{2 \cdot Q}{T} \cdot \frac{c_o}{c_s}} \cdot \sqrt{\frac{c_h}{c_h + c_s}} \quad (7)$$

The minimum total cost incurring during the whole period together with the money spent on purchased stocks (Stock & Lambert, 2001; Krampe et al., 2012; Vörös, 1991; Kulcsár, 1998):

$$C' = \sqrt{2 \cdot Q \cdot T \cdot c_o \cdot c_h \cdot \frac{c_s}{c_h + c_s}} + Q \cdot v \quad (8)$$

If the purchase price of the items does not change during the time and it is independent of the ordered volume, the annual purchase value can be considered as constant, so the equation can be simplified. The function of the total costs can be defined by replacing the time factors, and this relation will have a role in the future analysis:

$$C(q; d) = \frac{T}{t} \cdot c_o + \frac{t_1^2}{t^2} \cdot \frac{T \cdot q}{2} \cdot c_h + \frac{t_2^2}{t^2} \cdot \frac{T \cdot q}{2} \cdot c_s \rightarrow \min \tag{9}$$

where:

C – the total cost of inventory management for the examined period, without the value of purchased items.

The purchase cost of the analysed period can also be defined as follows:

$$C_o = \sqrt{\frac{Q \cdot T \cdot c_o \cdot c_h}{2}} \cdot \sqrt{\frac{c_s}{c_h + c_s}} \tag{10}$$

The cost of stock holding incurred during the whole analysed period can also be defined with full knowledge of the purchase cost as follows:

$$C_h = \frac{c_s}{c_h + c_s} \cdot \sqrt{\frac{Q \cdot T \cdot c_o \cdot c_h}{2}} \cdot \sqrt{\frac{c_s}{c_h + c_s}} = C_o \cdot \frac{c_s}{c_h + c_s} \tag{11}$$

The cost of stock shortage incurred during the whole analysed period can be defined with full knowledge of the purchase cost with the following relation:

$$C_s = \frac{c_h}{c_h + c_s} \cdot \sqrt{\frac{Q \cdot T \cdot c_o \cdot c_h}{2}} \cdot \sqrt{\frac{c_s}{c_h + c_s}} = C_o \cdot \frac{c_h}{c_h + c_s} \tag{12}$$

By replacing the above-mentioned relations in Formula (4) without the value of the purchased items, the total costs of the whole analysed period can also be defined with full knowledge of the purchase cost:

$$C = C_o + C_o \cdot \frac{c_s}{c_h + c_s} + C_o \cdot \frac{c_h}{c_h + c_s} = \tag{13}$$

$$= C_o \cdot \left(1 + \frac{c_s}{c_h + c_s} + \frac{c_h}{c_h + c_s} \right) = 2 \cdot C_o$$

Relations (4) and (13) show that in an optimal balanced state, the purchase cost represents half of the total costs, while the other half is the sum of the costs of the stock holding and stock shortage. The following relation can be derived:

$$\frac{C}{2} = C_o = C_h + C_s \tag{14}$$

3.2. RELATION BETWEEN COST FACTORS AND LEVEL LINES

In the case of the model allowing a stock shortage able to supplement a backlog in one batch, the whole stockpiling period can be divided into two periods, the one covered by stocks and the one with the stock shortage. The length of the periods is determined by the level of the specific cost projected onto the time units, and their proportion within the whole stockpiling period is given by the ratio of the two cost groups. The bigger the respective cost factor, the bigger is the extent of the occurrence period to be shortened. When the two cost factors are on the same level, their occurrence length is balanced as well, meaning that the alternating periods covered by stocks and with stock shortage have the same length.

The optimal cost level can be reached at the ratio where the different special costs together with their length of occurrence reflect the lowest level, thus, where the sum of $C_h + C_s$ is the lowest.

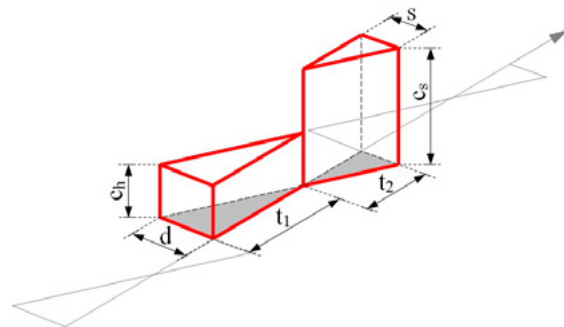


Fig. 2. Relation of holding and shortage costs and intervals within one period

The different specific cost factors and the period attributable to them can also be interpreted as a triangular-based prism, the bases of which are the stock level and the time profile of the backlog, while its height is given by the multiplier of the specific costs

attributable to the respective period. The balanced state of the volume of these two bodies is formed, where their joint volume has the smallest value (Fig. 2).

In case we quantify the different volumes, we can see that the result is identical with the formula for different stockpiling periods at the models with the stock shortage, for which we seek the minimum value:

$$\frac{1}{2} \cdot d \cdot t_1 \cdot c_h + \frac{1}{2} \cdot s \cdot t_2 \cdot c_s \rightarrow \min \quad (15)$$

The following equation shows the minimum stock holding and stock shortage for one stockpiling period:

$$\frac{1}{2} \cdot q \cdot t \cdot \left(\frac{c_s}{c_h + c_s} \right)^2 \cdot c_h + \frac{1}{2} \cdot q \cdot t \cdot \left(\frac{c_h}{c_h + c_s} \right)^2 \cdot c_s = \frac{1}{2} \cdot q \cdot t \cdot c_{(c_h; c_s)} \quad (16)$$

where:

$c_{(c_h; c_s)}$ – the optimal ratio of specific costs of stock holding and stock shortage projected on the stockpiling period.

The solution of the equation results in the multiplier, which shows the most favourable cost level during the stockpiling period:

$$c_{(c_h; c_s)} = \frac{c_h \cdot c_s}{c_s + c_h} \quad (17)$$

The significance of this relation is that during the modelling, the costs of stock holding and stock shortage can be replaced by a single multiplying factor, thus the analysis can be simplified to a basic model made up of one purchase and stock holding cost.

By assuming a consistent and continuous utilisation demand, this can also be interpreted as calculating a single volume from the volume attributed to the two periods, where the base of the triangular-based prism is the whole stockpiling period t and the quantity q , and its height is given by the multiplier (Fig. 3).

It is useful to show the cost factors as a function of one another. Figure 4 shows the specific cost of stock shortage in relation to the specific cost of stock holding. The diagram must be interpreted as showing the joint cost levels attributable to the different relations of these two specific cost factors. The thick red continuous curve shows the joint costs of stock holding and stock shortage by the respective specific cost levels c_h and c_s . The cost indicated with the continuous

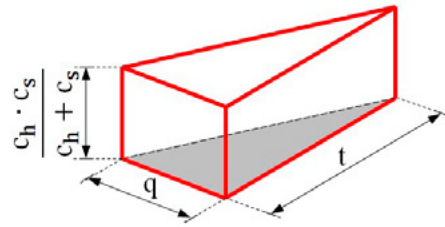


Fig. 3. Replacement of stock holding and stock shortage costs by a multiplying factor of a weighted specific cost

line reflects the same level at every point of the curve, which means that a given cost level in case of a higher stock holding cost factor can be reached only by a lower stock shortage cost factor. These convex curves with a negative slope show single level lines that reflect an increasing value when distancing from the initial point.

We can see that the cost curve in the direction of both specific cost factors is limited from below within the set of positive numbers. The function also has quantifiable values within the range below the limit value, but the result shows a negative value. The negative section of the function falls out of the domain since this would mean that we would have to calculate with negative stock holding and stock shortage costs, which cannot be interpreted in practice. The lower limit values also mean that optimisation can be realised only within a given range, which is the optimisation range. The lower limit values above zero also mean that by assuming a fixed quantity q , in the case of every $0 < c_h < 1$ there is a stock holding and stock shortage period with a certain length.

When defining relation (14), we saw that the total cost of stock management reaches its minimum where the purchase cost is identical with the joint costs of stock holding and stock shortage. This also proves that every point of the continuous red curve is identical in the case of the given level line with the half of stockpiling cost C attributable to a defined quantity q , which results from $C = C_o + C_h + C_s$ and $C_{hs} = C_h + C_s$ and $C_{(min)} = C_{hs} = C_o$.

The initial condition of the model is that factors q , Q , T , and c_o are fixed values, which means that the value t is also fixed. Thus, only cost factors c_h and c_s , levels d and s , and the length of time periods t_1 and t_2 can be changed. Since values c_o , Q and q are constant, the derivable purchase cost C_o is constant too, which, in optimal cases, is identical with the half of all stockpiling costs, an equality that was proved with relation (13). The definition of the minimum level of total costs is the task.

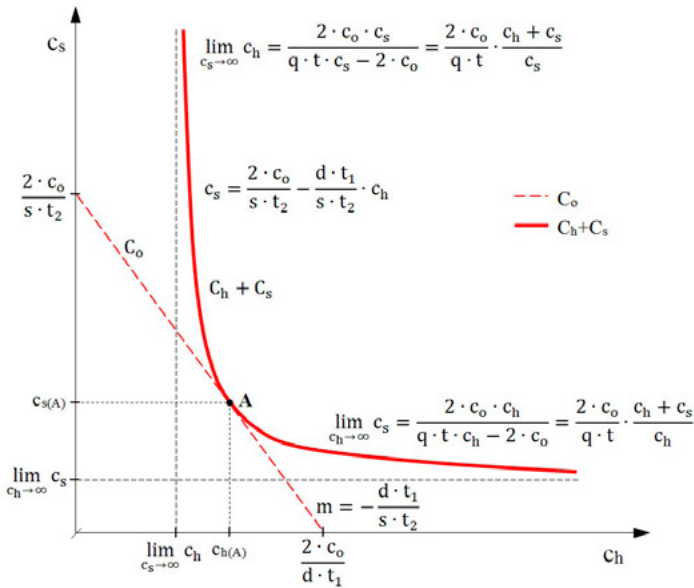


Fig. 4. Relation between cost factors and the level line, representation of the optimal point and of limit values

The costs related to purchases can be drawn onto the diagram as well. The dotted red line shows the purchase cost C_o , which, in the case of a fixed q value is independent of factors c_h and c_s . The line shows an identical level at every point. Its slope is the result of the ratio of the average stock level and the average backlog quantifiable during the period of stock holding and stock shortage.

In an optimal balance state, the purchase cost is identical to the sum of stock holding and stock shortage costs.

$$\frac{Q}{q} \cdot c_o = \frac{1}{2} \cdot d \cdot t_1 \cdot \frac{Q}{q} \cdot c_h + \frac{1}{2} \cdot s \cdot t_2 \cdot \frac{Q}{q} \cdot c_s \quad (18)$$

By rearranging the equation, we can get the following equality, which shows the cost of stock shortage c_s in relation to stock holding c_h :

$$c_s = \frac{2 \cdot c_o}{s \cdot t_2} - \frac{d \cdot t_1}{s \cdot t_2} \cdot c_h \quad (19)$$

The slope of purchase cost line C_o can be directly defined by this formula:

$$m = -\frac{d \cdot t_1}{s \cdot t_2} \quad (20)$$

where:

m – the slope of the purchase cost.

The points of intersection of the purchase cost axis can be defined from Formula (19); thus, the purchase function can be shown with knowledge of the slope and the points of intersection:

$$\text{Vertical axial section: } \frac{2 \cdot c_o}{s \cdot t_2} \quad (21)$$

$$\text{Horizontal axial section: } \frac{2 \cdot c_o}{d \cdot t_1} \quad (22)$$

The optimal total costs can be given using the relation $C=C_o+C_h+C_s$ and in balanced state with $C_o=C_h+C_s$ and the result of rearranging the two formulae, as proofed in the Formula (13):

$$C = 2 \cdot C_o \quad (23)$$

The equality can be given by replacement as follows:

$$\sqrt{2 \cdot Q \cdot T \cdot c_o \cdot c_h \cdot \frac{c_s}{c_h + c_s}} = 2 \cdot \frac{Q}{q} \cdot c_o \quad (24)$$

Within a positive domain, Formula (24) allows us to deduct the lower limit values of the different specific cost factors; these fix the position of curve C_h+C_s and close the optimisation range:

$$\lim_{c_s \rightarrow \infty} c_h = \frac{2 \cdot c_o \cdot c_s}{q \cdot t \cdot c_s - 2 \cdot c_o} = \frac{2 \cdot c_o}{q \cdot t} \cdot \frac{c_h + c_s}{c_s} \quad (25)$$

$$\lim_{c_h \rightarrow \infty} c_s = \frac{2 \cdot c_o \cdot c_h}{q \cdot t \cdot c_h - 2 \cdot c_o} = \frac{2 \cdot c_o}{q \cdot t} \cdot \frac{c_h + c_s}{c_h} \quad (26)$$

The diagram behaves similarly to the indifference curves and the budget line known from microeconomics (Kopányi, 1996; Böventer, 1991), but we must emphasise that in practice, there is no substitutability between the specific cost of stock holding c_h and the specific cost of stock shortage c_s , since the modification, e.g. increase of a cost factor will not result in the decrease of the other; within an optimisation, only the ratio of periods t_1 and t_2 will shift in one direction. The significance of the model is that it represents the level line of costs, the movement of the different cost factors in relation to others, and their operating mechanism; thus, it facilitates the representation of costs and the definition of the direction of optimisation.

In case factors c_h and c_s keep their original ration and increase from the point $c_{h(A)};c_{s(A)}$ to the point $c_{h(B)};c_{s(B)}$, the value of q , by a higher cost level C_h+C_s , will remain unaltered. Figure 5 shows the shifting

between the level lines when distancing from the initial point. Each point of the dotted straight line drawn from the initial point reflects a similar ratio of specific costs c_h and c_s , the further from the initial point the higher the cost levels shown by the level lines, thus the points of intersection of the straight line and of the different level lines show the proportionate changes in specific costs c_h and c_s .

In a balanced state, a purchase cost line $C_{o(B)}$ can be drawn to the higher level line, the slope of which remains identical with the slope of the line $C_{o(A)}$ due to the invariability of the $c_h;c_s$ ratio. In case the two specific cost factors become more expensive, their impact on purchase would result in more frequent purchases by smaller q quantities. Since quantity q was fixed among the initial conditions, a purchase line drawn to a higher-level line in a balanced state can be drawn only by a higher one-off purchase cost c_o . However, the alteration of $c_h;c_s$ in practice does not influence the one-off purchase cost c_o , making it clear that the balanced state does not reflect an optimal state. To achieve the optimal state, we must lift the fixedness of q , and the order quantity could be optimised by a new q value. In practice, however, we could face the situation when the fixedness of q cannot be lifted, e.g. the deliveries cannot be organised to be more frequent. This state cannot be considered as an optimal one.

Another interpretation of Figure 5 is that instead of a proportionally changing a specific stock holding cost c_h and a specific stock shortage cost c_s , it is the one-off purchase cost c_o that changes. In case the one-off purchase cost c_o increases, it would result in an increased purchase cost $C_{o(B)}$ by an unaltered purchase frequency. This can be reduced by making the purchases less frequent, which would lead to the increase in delivery quantities and the increase in the average level of stocks. Since the one-off purchase cost c_o does not influence factors $c_h;c_s$, the modification of c_o affects the joint level of costs C_h+C_s only through the q purchase quantities. In case quantity q is fixed, we can see that the new state is not optimal, since the frequency of purchases must be changed for optimisation, which cannot be realised due to the fixedness of quantity q .

In the case of a change of only one of the specific costs c_h and c_s , the ratio of time periods t_1 and t_2 will change as well. A shift is not possible on the curve C_h+C_s representing a similar cost level since the shift on the curve could be achieved only by a shift in the opposite direction of the other cost factor. However, since these two specific cost factors do not replace each other in practice, the alteration of one factor does not cause the shift of the other factor in the opposite direction. Thus, costs C_h+C_s will show a new level line in this situation. In case the value of the specific cost factor increases, the distance between the level line and the initial point grows (Figure 6).

The line of the purchase cost $C_{o(B)}$ can also be drawn as a tangent line to the points $c_{h(B)};c_{s(B)}$ of the new level line. The level line and the tangent point of the line give the optimum to the new values $c_{h(B)};c_{s(B)}$. In case only one of factors c_h and c_s changes, or both do in a way that their ratio changes as well in a certain direction, the slope of the purchase cost $C_{o(B)}$ drawable to the curve indicates the new level change. This is the result of the fact that the ratio of time periods t_1 and t_2 is rearranged due to the shifting of the specific cost of stock holding from the specific cost of the backlog; thus, the amount of optimum order quantity will change for the whole period. To draw the tangent line to the new level line, we must lift the fixedness of q among the initial condition, otherwise, the purchase cost $C_{o(B)}$ would remain unaltered, which must be made equal to the new cost level to achieve balance.

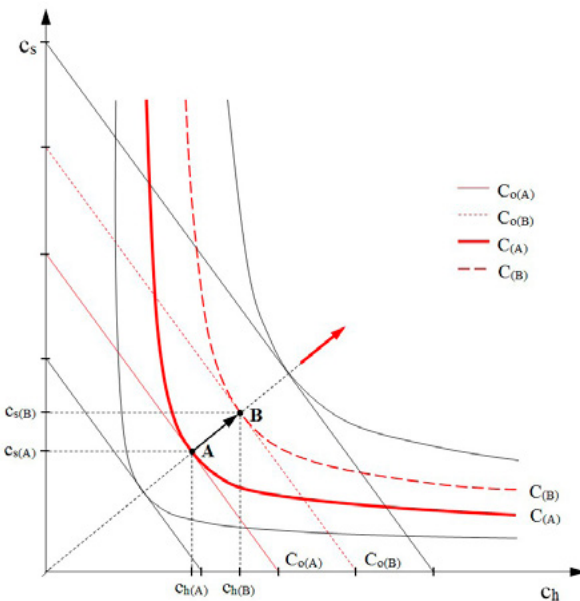


Fig. 5. Shift of the level of the total costs in the case of a proportionate change of the specific stock holding and stock shortage costs

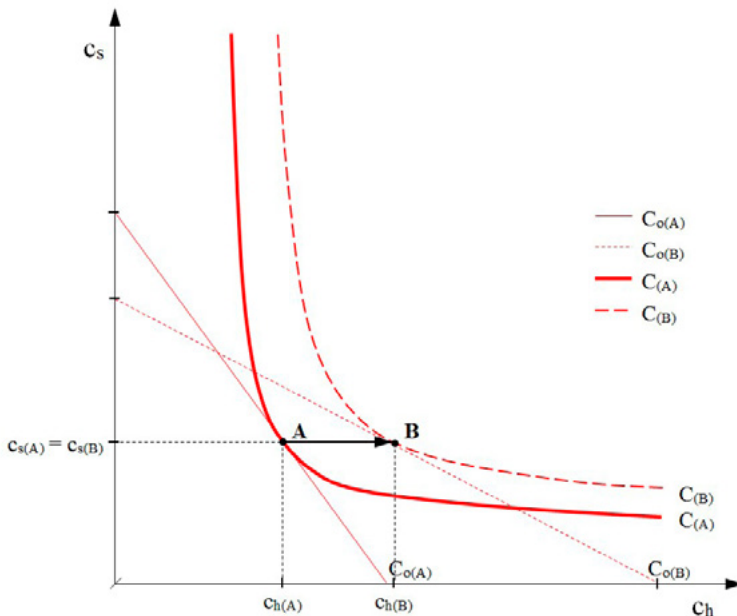


Fig. 6. Modification of the level line of total costs in the case of an alteration of the specific stock holding cost

CONCLUSIONS

It often happens in practical logistics that either the ordering cost or the inventory holding cost and stock-out cost change with the time. In each case, it is necessary to draw the purchase line and the positions of cost factors c_h and c_s to examine the initial point and to discover the possibility of optimisation. In case these do not coincide, the initial state does not reflect an optimal state. Optimisation must be carried out with knowledge of the modifiable parameters and along the described operating mechanism. In case one has fixed a single factor among the factors necessary for optimisation, the optimum cannot be reached.

By using the described model, we can define the optimisation range of the different cost factors of the inventory management system, and also the direction of the optimisation. The model represents the level line of costs, the movement of the different cost factors in relation to others and their operating mechanism.

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